

Modelling practical certainty and its link with classical propositional logic

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Introduction
Context: A subject who is practically certain about the occurrence of every event in a collection \mathcal{F} .
Accept & reject statements: We try to model this certainty in the language of accept and reject statement-based uncertainty models.
Motivation: This language is rich enough to encompass the different approaches of Walley and de Finetti. In order to obtain more insight in these approaches, we study different types of assessments.
Set of events: The random variable \mathcal{F} about which the subject expresses practical certainty takes values in \mathcal{F} . All events are collected in the power set $\mathcal{P} = \{A, A^c, \mathcal{F}\}$. $\mathcal{B}_A \subseteq \mathcal{P}$ is a filter base if A is closed under finite intersections (closed under \cap), $A, B \in \mathcal{B}_A \Rightarrow A \cap B \in \mathcal{B}_A$. Then the $A, B \in \mathcal{F}$ is called proper if in addition $B \cap A^c \in \mathcal{B}_A$. \mathcal{F} is called a filter if: (i) \mathcal{F} is closed under union, and (ii) \mathcal{F} is increasing (closed under finite unions). If $A \in \mathcal{F}$ and $A \supseteq B$, then also $B \in \mathcal{F}$. \mathcal{F} is called proper if in addition $B \in \mathcal{F}$. We denote the set of all proper filters by \mathcal{F} .
Set of gambles: A gamble f is a bounded real-valued function on \mathcal{F} . The set of all gambles is \mathcal{G} . $f: \mathcal{F} \rightarrow \mathbb{R}$ is for all $x \in \mathcal{F}$, we write $f(x) = f$, and the set of all such gambles is \mathcal{G} . We write $f \geq 0$ if and only if $f(x) \geq 0$ for all $x \in \mathcal{F}$, and the set of all such gambles is \mathcal{G}_+ . If $f(x) = 0$ for all $x \in \mathcal{F}$, we write $f = 0$, and the set of all such gambles is \mathcal{G}_0 .

First type: favourability assessment
Favourability: A gamble f is favourable if $f \in \mathcal{G}_+$ and $f \neq 0$.
Accepted and / or rejected:
Assessment about one event A : If a subject is practically certain that an event A occurs, we will take this to mean that he finds any gamble in $\mathcal{G}_+ \cap \{0, -1, +1\}$ favourable; he accepts to bet on A at odd $+1$, and refuses to bet against A at odds -1 .
Assessment about more events \mathcal{F} : If he is practically certain that each event in \mathcal{F} occurs, then his assessment is
 $\mathcal{G}_+ \cap \{0, -1, +1\}$ with $\mathcal{G}_+ \cap \{0, -1, +1\} \cap \mathcal{F} \neq \emptyset$.

Background model: We assume the background model $\mathcal{P} = \{0, \mathcal{F}, \mathcal{F}^c\}$, which yields the smallest assessment $\mathcal{B} = \mathcal{P} \cap \mathcal{F}$ that includes both \mathcal{F} and \mathcal{F}^c .

Deductive closure: The deductively closed assessment $\mathcal{D} = \text{cl}_{\mathcal{D}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid g \geq 0\}$ is determined by
 $\text{cl}_{\mathcal{D}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$
 where $\mathcal{F} = \{A_1, \dots, A_n \mid c \in \mathbb{N}_+, A_i \in \mathcal{F}\} \subseteq \mathcal{P}$ is the filter base generated by \mathcal{F} .
 $\mathcal{D} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$

No Confusion: It is equivalent with the finite intersection property:
 $\mathcal{D} \neq \emptyset$ if and only if $\mathcal{F} \neq \emptyset$ or, equivalently, if $\mathcal{F} \neq \emptyset$.

No Limbo: The no-lingering extension $\mathcal{E} = \text{cl}_{\mathcal{E}} \mathcal{B}$ is given by
 $\text{cl}_{\mathcal{E}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$
 so $\mathcal{E} = \mathcal{D} \cup \mathcal{F}^c$.
 The last two examples have No Confusion, what means that the expected practical certainty is not in question. Therefore, these examples are not continued.

All practical certain events: Check the inference procedure described above, which shows us a filter from the set of favourable gambles. The larger set of favourable gambles, \mathcal{G}_+ , bear any relationship to inference in classical propositional logic? For which events $A \in \mathcal{F}$, $B \in \mathcal{F}$, $C \in \mathcal{F}$?

This tells us that our specific interpretation of the logic of practical certainty has the same basic machinery as classical propositional logic.

Accept & reject statements
Accepting & rejecting: The subject gives his assessment \mathcal{D} by making accept and reject statements about gambles $f \in \mathcal{G}$.
Accepting f : implies a commitment for the subject to engage in the following transaction: (i) he gives the actual value of f is determined. (ii) he gets the possible negative $-|g(f)|$.
Rejecting f : means that the subject excludes f from being accepted.
Assessment \mathcal{D} : is a pair of accepted $\{+\}$ and rejected $\{-\}$ gambles: $\mathcal{D} = \{g \in \mathcal{G}_+, h \in \mathcal{G}_-\}$.

Second rationality requirement:
 \mathcal{D} should be deductively closed: $\text{cl}_{\mathcal{D}} \mathcal{B} \subseteq \mathcal{D}$.

No Confusion: Claims the interpretation attached to an accept and a reject statement, we have as a third rationality requirement:
 No-lingering: $\mathcal{D} \neq \emptyset$ if and only if $\mathcal{F} \neq \emptyset$.

Fourth rationality requirement:
 \mathcal{D} should have no limbo: $\mathcal{D} \cap \mathcal{F}^c = \emptyset$.

Background model: Before an assessment is given, some gambles can be pre-empted to be accepted others to be rejected. Such a priori assumption can be captured by posing a background model \mathcal{P} .

First rationality requirement:
 \mathcal{D} is determined by $\mathcal{B} = \mathcal{P} \cap \mathcal{F}$.

Deductive closure: If f and g are acceptable, then so should be $f \vee g$ and also $f \wedge g$. These two observations are summarised in the deductive extension
 $\mathcal{D} = \text{cl}_{\mathcal{D}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid g \geq 0\}$,
 where $\text{cl}_{\mathcal{D}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$, the positive linear hull of \mathcal{B} .

Second type: indifference assessment
Indifference: A gamble f is indifferent if $f \in \mathcal{G}_+$ and $f \in \mathcal{G}_-$, both f and to rejection $-f$ are accepted.
Assessment about one event A : If a subject is practically certain that an event A occurs, we will now take this to mean that he is indifferent between 1 and -1 , or equivalently between 1 and 0 .
Assessment about more events \mathcal{F} : If he is practically certain that each event $\mathcal{F} \subseteq \mathcal{F}$ occurs, then his assessment is
 $\mathcal{G}_+ \cap \mathcal{G}_- \cap \{0, -1, +1\}$ with $\mathcal{G}_+ \cap \mathcal{G}_- \cap \{0, -1, +1\} \cap \mathcal{F} \neq \emptyset$.

Background model: Because $\mathcal{A}_0 \subseteq \mathcal{F}$, the nature of the assessment forces us to assume a slightly different background model: $\mathcal{P} = \{0, \mathcal{F}, \mathcal{F}^c\}$. This yields the smallest assessment $\mathcal{B} = \mathcal{P} \cap \mathcal{F}$ that includes \mathcal{F} and \mathcal{F}^c .

Deductive closure: The deductively closed assessment $\mathcal{D} = \text{cl}_{\mathcal{D}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid g \geq 0\}$ is determined by
 $\text{cl}_{\mathcal{D}} \mathcal{B} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$
 so the different gambles are
 $\mathcal{D} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$.

Embedding classical propositional logic
Indifference assessments
 Denote the collection of all assessments by \mathcal{A} . Consider the family of models for practical certainty following from indifference assessments:
 $\mathcal{C} = \{(\mathcal{F}, \mathcal{D}) \mid \mathcal{F} \subseteq \mathcal{A}, \mathcal{D} \neq \emptyset\}$.
 Then we have
 $(\mathcal{A}, \mathcal{C})$ is a strong belief structure,
 meaning that $(\mathcal{A}, \mathcal{C})$ is a complete lattice where \top plays the role of infimum, \emptyset is an intersection operation, for any $A \in \mathcal{A}$, $B \in \mathcal{C}$, $\text{cl}_{\mathcal{D}} \{A, B\} \subseteq \mathcal{C}$ has no top, and $\text{cl}_{\mathcal{D}} \{A, B\} \subseteq \mathcal{C}$ is dually atomic: $\mathcal{C} \cap \emptyset$ and $\mathcal{D} = \{g \in \mathcal{G}_+ \mid \exists \lambda \in \mathbb{N}_+, \exists c \in \mathbb{N}_+, \exists \mathcal{F} \subseteq \mathcal{F} \text{ s.t. } g \geq c \cdot \mathcal{F}\}$ if $\mathcal{C} \neq \emptyset$. We have also
 $\mathcal{D} \in \mathcal{P}$, $\{0, \mathcal{F}, \mathcal{F}^c\} \in \mathcal{A}$ is order isomorphic,
 which means that \mathcal{F} and \mathcal{C} are essentially the same.

Favourability assessments
 Consider the family of models for practical certainty following from favourability assessments:
 $\mathcal{C} = \{(\mathcal{F}, \mathcal{D}) \mid \mathcal{F} \subseteq \mathcal{A}, \mathcal{D} \neq \emptyset, \mathcal{D} \cap \mathcal{F}^c = \emptyset\}$.
 Unfortunately, $(\mathcal{A}, \mathcal{C})$ is not a strong belief structure:
 $(\mathcal{A}, \mathcal{C})$ is not an intersection structure.
 Luckily, we can still find an embedding of \mathcal{F} into \mathcal{C} . Consider a coherent set of favourable gambles \mathcal{G}_+ , derived from an assessment that includes \mathcal{F} and take any $f \in \mathcal{G}_+$ such that $\mathcal{D} \cap \mathcal{F}^c = \emptyset$. Let
 $\mathcal{D} \in \mathcal{P}$, $\{0, \mathcal{F}, \mathcal{F}^c\} \in \mathcal{A}$. Then
 $\mathcal{D} \in \mathcal{P}$, $\{0, \mathcal{F}, \mathcal{F}^c\} \in \mathcal{A}$, $\mathcal{D} \cap \mathcal{F}^c = \emptyset$.

Introduction

Introduction

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Introduction

General A **belief** tells us practically certain about the occurrence of every event ω in a domain Ω .

Language & logical statements. We try to model this certainty in the form of **accept** and **reject** statements based uncertainty models.

Modeling. The long step is to establish an equivalence between the different approaches of Walley and de Finetti. In order to obtain more insight in these approaches, we study different types of assessments.

State of events. The random variable \mathcal{A} about which the subject accepts (rejects) precisely their values in \mathcal{P} . All events are collected in the power set $\mathcal{P} = \{A, A^c, \emptyset, \Omega\}$. \mathcal{A} is \mathcal{P} -A and \mathcal{A}^c is called **state of event** under the interpretation classical-order interpretation. If $\mathcal{A} = \mathcal{P}$, then also $\mathcal{A}^c = \mathcal{P}$. \mathcal{A} is called **proper** if in addition $\mathcal{A} \neq \mathcal{P}$. \mathcal{A} is called **under** (over) **improper** (proper) if its underlying object under (over) **improper** (A) $\neq \mathcal{P}$ and \mathcal{A} . Then also \mathcal{A}^c is called **proper** (under) **improper** if \mathcal{A} . We denote the set of all proper (under) \mathcal{P} (states of) **games**. A **game** \mathcal{A} is a bounded real-valued function on \mathcal{P} . The set of all **games** is \mathcal{G} . We write $\mathcal{A} \in \mathcal{G}$ or $\mathcal{A} \in \mathcal{G}$. We write $\mathcal{A} \in \mathcal{G}$ if and only if all **games** is \mathcal{G} .

Assessing. A **game** \mathcal{A} is **assessed** if $\mathcal{A} \in \mathcal{G}$ and $\mathcal{A} \in \mathcal{G}$. Assessment and \mathcal{A} is **finite**.

Assessment about one event. If a subject is practically certain that no event ω occurs, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} . In fact, if $\mathcal{A} \in \mathcal{G}$, then $\mathcal{A} \in \mathcal{G}$. In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} . In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} .

Assessment about many events. If a subject is practically certain that each event in \mathcal{P} occurs, then the assessment is

Deductive closure. The deductively closed assessment $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$ is determined by

where $\mathcal{A}(\omega) = \mathcal{A}(\{ \omega \})$ and $\mathcal{A}(\omega) = \mathcal{A}(\{ \omega \})$. The first line generated by \mathcal{A} .

No Contradiction. \mathcal{A} is equivalent with this restriction property.

No Limits. The following extension $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$ is given by

All practical certain events. One the inference property is observed, which allows us to infer from the set of decidable games \mathcal{A} , the larger set of all **practical certain** events. One the inference property is observed, which allows us to infer from the set of decidable games \mathcal{A} , the larger set of all **practical certain** events. One the inference property is observed, which allows us to infer from the set of decidable games \mathcal{A} , the larger set of all **practical certain** events.

This tells us that an explicit interpretation of the type of practical certainty has the same basic modelling an classical propositional logic.

Assessing & rejecting. The random game \mathcal{A} is assessed (not) by making **accept** and **reject** statements about games \mathcal{A} .

Accepting. implies a commitment for the subject to accept in the first instance.

Rejecting. implies a commitment for the subject to reject in the first instance.

Assessment \mathcal{A} . is a part of accepted \mathcal{A} and rejected \mathcal{A} (games $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$).

Assessing & rejecting. The random game \mathcal{A} is assessed (not) by making **accept** and **reject** statements about games \mathcal{A} .

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Assessment \mathcal{A} . is a part of accepted \mathcal{A} and rejected \mathcal{A} (games $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$).

There are four reliability requirements.

Background model. Before an event ω is given, some games can be given to be assessed and others to be rejected. But a given assessment can be captured by a background model \mathcal{A} .

Full reliability requirement.

No Limits. One can still accept or reject each **improper** game $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$. Games \mathcal{A} are assessed when $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$ in the model \mathcal{A} . The observation is interpreted in the following extension

Full reliability requirement.

Deductive closure. If \mathcal{A} and \mathcal{B} are accepted, then so is $\mathcal{A} \cup \mathcal{B}$ and $\mathcal{A} \cap \mathcal{B}$. These operations form are summarized in the following extension

$\mathcal{A} \cup \mathcal{B} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \text{ or } \mathcal{B}(\omega) = 1 \}$

where $\mathcal{A} \cup \mathcal{B} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \text{ or } \mathcal{B}(\omega) = 1 \}$ and $\mathcal{A} \cap \mathcal{B} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \text{ and } \mathcal{B}(\omega) = 1 \}$. The picture here hold \mathcal{A} .

Assessing. A **game** \mathcal{A} is **assessed** if $\mathcal{A} \in \mathcal{G}$ and $\mathcal{A} \in \mathcal{G}$. Assessment and \mathcal{A} is **finite**.

Assessment about one event. If a subject is practically certain that no event ω occurs, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} . In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} .

Assessment about many events. If a subject is practically certain that each event in \mathcal{P} occurs, then the assessment is

Background model. Because \mathcal{A} is finite, the values of this assessment form can be written a slightly different background model. \mathcal{A} is the first line generated by \mathcal{A} . This gives the extended assessment $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$.

Deductive closure. The deductively closed assessment $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$ is determined by

where $\mathcal{A}(\omega) = \mathcal{A}(\{ \omega \})$ and $\mathcal{A}(\omega) = \mathcal{A}(\{ \omega \})$. The first line generated by \mathcal{A} .

All practical certain events. To test all such events, we test all the events \mathcal{A} in \mathcal{G} for which we have $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$. We test

which leads to the same conclusions as for the first type of assessment.

Preference assessments

Denote the collection of all assessments by \mathcal{A} . Consider the family of models for practical certainty following from preference assessments

$\mathcal{A} = \{ \mathcal{A} \in \mathcal{G} : \mathcal{A} \in \mathcal{G} \text{ and } \mathcal{A} \in \mathcal{G} \}$

This is the

where $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$ is a complete lattice when \mathcal{A} is the set of all events \mathcal{A} in \mathcal{G} or \mathcal{A} is the set of all events \mathcal{A} in \mathcal{G} . For any $\mathcal{A} \in \mathcal{G}$, $\mathcal{A} \in \mathcal{G}$ and $\mathcal{A} \in \mathcal{G}$, we have $\mathcal{A} \in \mathcal{G}$ and $\mathcal{A} \in \mathcal{G}$. In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} . In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} .

Formability assessments

Denote the family of models for practical certainty following from formability assessments

$\mathcal{A} = \{ \mathcal{A} \in \mathcal{G} : \mathcal{A} \in \mathcal{G} \text{ and } \mathcal{A} \in \mathcal{G} \}$

where $\mathcal{A} = \{ \omega \in \Omega : \mathcal{A}(\omega) = 1 \}$ is a complete lattice when \mathcal{A} is the set of all events \mathcal{A} in \mathcal{G} or \mathcal{A} is the set of all events \mathcal{A} in \mathcal{G} . For any $\mathcal{A} \in \mathcal{G}$, $\mathcal{A} \in \mathcal{G}$ and $\mathcal{A} \in \mathcal{G}$, we have $\mathcal{A} \in \mathcal{G}$ and $\mathcal{A} \in \mathcal{G}$. In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} . In fact, we will not use this to mean that he finds any game \mathcal{A} in \mathcal{G} .

Accept & reject statements

Accept & reject statements

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Context: A subject who is practically certain about the occurrence of every event in a collection \mathcal{E} .
Accept & reject statements: We try to model this certainty in the form of accept and reject statements based on elementary events.
Modeling: This language is rich enough to accommodate the obvious fact that the language is not closed under negation.
Assessment: Three operations, we study different types of assessments.
Practical certainty: This notion is used to model the subject's practical certainty about events in \mathcal{E} . All events are covered in the sense that $\mathcal{A} \cup \mathcal{R} = \mathcal{E}$.
Practical consistency: This notion is used to model the subject's practical consistency about events in \mathcal{E} .
Practical coherence: This notion is used to model the subject's practical coherence about events in \mathcal{E} .
Practical sufficiency: This notion is used to model the subject's practical sufficiency about events in \mathcal{E} .
Practical necessity: This notion is used to model the subject's practical necessity about events in \mathcal{E} .

Assessment: A gamble β is desirable if $\beta \succcurlyeq 0$, i.e., $\beta \succcurlyeq 0$ is the assessment of β .
Assessment about an event: If a subject is practically certain that an event A occurs, we will not give her the bet that she has any gain from A^c and no loss from A .
Assessment about a gamble: If a subject is practically certain that each event $\beta \succcurlyeq 0$, then her assessment is β .
Background model: We assume the background model $\mathcal{M} = (\mathcal{E}, \mathcal{P})$, which yields the model assessment $\beta = 0$ if $\beta \succcurlyeq 0$ and $\beta \preccurlyeq 0$ if $\beta \preccurlyeq 0$.

Deductive closure: The deductively closed assessment $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$ is determined by β .
Line: $\beta = \lambda \beta_1 + (1-\lambda)\beta_2$, the line generated by β_1 and β_2 .
No Corollary: An equivalent with the Side condition property.
No Links: The following extension $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$ is given by $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$.

All practical certain events: Shows the inference procedure described above, which yields a set from the set of basic gambles, as the larger set of favorable gambles \mathcal{A} . After any reasoning to inference is closed, practical certainty is closed.
Line: $\beta = \lambda \beta_1 + (1-\lambda)\beta_2$, the line generated by β_1 and β_2 .
Side condition: This side condition is a specific interpretation of the Side condition property from the above basic machinery to classical propositional logic.

Accept & reject statements

Accepting & rejecting: The subject gives his assessment of β by saying accept and reject statements about gambles $\beta \succcurlyeq 0$.
Accepting $\beta \succcurlyeq 0$: implies a commitment for the subject to sign in the following transaction:
 - She will value β if β is a gamble.
 - If she gets the positive (negative - payoff) β .
Rejecting $\beta \preccurlyeq 0$: means that the subject is not willing to sign being accepted.
Assessment of β : is a pair of accepted ($\beta \succcurlyeq 0$) and rejected ($\beta \preccurlyeq 0$) gambles $\beta = (\beta \succcurlyeq 0, \beta \preccurlyeq 0)$.

Second rationality requirement:
No Corollary: Given the integration attached to an assessment and a reject statement, we have as a third rationality requirement:
 - This rationality requirement is captured by a background model \mathcal{M} .

Then are four rationality requirements.

Background model: Before an assessment is given, some gambles can be purchased to be accepted and others to be rejected. Each a prior assumption can be captured by posting a background model \mathcal{M} .

For rationality requirement:
 - This rationality requirement is captured by a background model \mathcal{M} .

Deductive closure: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical consistency: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical coherence: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical sufficiency: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical necessity: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.

Indifference: A gamble β is indifferent if $\beta \succcurlyeq 0$ and $\beta \preccurlyeq 0$.
Assessment about an event: If a subject is practically certain that an event A occurs, we will not give her the bet that she has any gain from A^c and no loss from A .
Assessment about a gamble: If a subject is practically certain that each event $\beta \succcurlyeq 0$, then her assessment is β .
Background model: We assume the background model $\mathcal{M} = (\mathcal{E}, \mathcal{P})$, which yields the model assessment $\beta = 0$ if $\beta \succcurlyeq 0$ and $\beta \preccurlyeq 0$ if $\beta \preccurlyeq 0$.
Deductive closure: The deductively closed assessment $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$ is determined by β .
Line: $\beta = \lambda \beta_1 + (1-\lambda)\beta_2$, the line generated by β_1 and β_2 .
No Corollary: An equivalent with the Side condition property.
No Links: The following extension $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$ is given by $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$.

Indifference assessments:
 Describe the collection of all assessments β . Consider the family of models for practical certainty following from indifference assessments.
Then we have:
 - This family is a convex set.
 - This family is a convex set.
 - This family is a convex set.
Practical certainty: This notion is used to model the subject's practical certainty about events in \mathcal{E} .
Practical consistency: This notion is used to model the subject's practical consistency about events in \mathcal{E} .
Practical coherence: This notion is used to model the subject's practical coherence about events in \mathcal{E} .
Practical sufficiency: This notion is used to model the subject's practical sufficiency about events in \mathcal{E} .
Practical necessity: This notion is used to model the subject's practical necessity about events in \mathcal{E} .

Accept & reject statements

Accepting & rejecting: The subject gives his assessment of β by saying accept and reject statements about gambles $\beta \succcurlyeq 0$.
Accepting $\beta \succcurlyeq 0$: implies a commitment for the subject to sign in the following transaction:
 - She will value β if β is a gamble.
 - If she gets the positive (negative - payoff) β .
Rejecting $\beta \preccurlyeq 0$: means that the subject is not willing to sign being accepted.
Assessment of β : is a pair of accepted ($\beta \succcurlyeq 0$) and rejected ($\beta \preccurlyeq 0$) gambles $\beta = (\beta \succcurlyeq 0, \beta \preccurlyeq 0)$.

Second rationality requirement:
No Corollary: Given the integration attached to an assessment and a reject statement, we have as a third rationality requirement:
 - This rationality requirement is captured by a background model \mathcal{M} .

Then are four rationality requirements.

Background model: Before an assessment is given, some gambles can be purchased to be accepted and others to be rejected. Each a prior assumption can be captured by posting a background model \mathcal{M} .

For rationality requirement:
 - This rationality requirement is captured by a background model \mathcal{M} .

Deductive closure: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical consistency: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical coherence: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical sufficiency: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.
Practical necessity: If β and γ are acceptable, then so should be $\beta \vee \gamma$ and $\beta \wedge \gamma$, with $\beta \vee \gamma = \text{acc}(\beta) \cup \text{acc}(\gamma)$ and $\beta \wedge \gamma = \text{rej}(\beta) \cup \text{rej}(\gamma)$.

Indifference: A gamble β is indifferent if $\beta \succcurlyeq 0$ and $\beta \preccurlyeq 0$.
Assessment about an event: If a subject is practically certain that an event A occurs, we will not give her the bet that she has any gain from A^c and no loss from A .
Assessment about a gamble: If a subject is practically certain that each event $\beta \succcurlyeq 0$, then her assessment is β .
Background model: We assume the background model $\mathcal{M} = (\mathcal{E}, \mathcal{P})$, which yields the model assessment $\beta = 0$ if $\beta \succcurlyeq 0$ and $\beta \preccurlyeq 0$ if $\beta \preccurlyeq 0$.
Deductive closure: The deductively closed assessment $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$ is determined by β .
Line: $\beta = \lambda \beta_1 + (1-\lambda)\beta_2$, the line generated by β_1 and β_2 .
No Corollary: An equivalent with the Side condition property.
No Links: The following extension $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$ is given by $\beta = \text{acc}(\beta) \cup \text{rej}(\beta)$.

Indifference assessments:
 Describe the family of models for practical certainty following from indifference assessments.
Then we have:
 - This family is a convex set.
 - This family is a convex set.
 - This family is a convex set.
Practical certainty: This notion is used to model the subject's practical certainty about events in \mathcal{E} .
Practical consistency: This notion is used to model the subject's practical consistency about events in \mathcal{E} .
Practical coherence: This notion is used to model the subject's practical coherence about events in \mathcal{E} .
Practical sufficiency: This notion is used to model the subject's practical sufficiency about events in \mathcal{E} .
Practical necessity: This notion is used to model the subject's practical necessity about events in \mathcal{E} .

Accept & reject statements

There are four rationality criteria.

Modelling practical certainty and its link with classical propositional logic
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Context. A subject who is practically certain about the occurrence of every element in collection Ω .

Accept & reject statements. We try to model this certainty by the concept of accept and reject statements based on classical propositional logic. This language is not enough to encompass the observed behavior. We study different types of extensions.

Assessment. We study different types of assessments.

Practical certainty. We study about the subject's own process/practical certainty about events in Ω . All events are covered in the sense that $\omega \in \Omega$ if and only if ω is in the support of the subject's process/practical certainty (event ω is not covered when there is no statement related with ω), $\omega \in \Omega$ if and only if ω is covered under negation and $\omega \notin \Omega$ is covering closed under the negation, $\omega \in \Omega$ if and only if ω is covered under \neg and $\omega \in \Omega$ if and only if ω is covered under \wedge . We describe the set of all events that are in a state of practical certainty. A gamble is a described bet function on Ω . The set of all gambles is $\mathcal{G} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0 \text{ for all } \omega \in \Omega\}$. We write $g \geq 0$ and $g \geq 0$ for each gamble g . $g \geq 0$ if and only if $g(\omega) \geq 0$ for all $\omega \in \Omega$. We write $g \geq 0$ and $h \geq 0$ for each gamble g, h .

Assessment. A gamble g is desirable if $g \geq 0$ and $g \geq 0$ is accepted and $g \geq 0$ is rejected.

Assessment about one event ω . If the subject is practically certain that an event ω occurs, we will not take this to mean that he finds any gamble in $\mathcal{G} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0\}$ desirable. He accepts to bet on ω if and only if he is not practically certain to win.

Assessment about three events $\omega_1, \omega_2, \omega_3$. If the subject is practically certain that each event ω_i occurs, then his assessment is

Background model. We assume the background model $\mathcal{P} = \{p : \Omega \rightarrow \mathbb{R} \mid p(\omega) \geq 0 \text{ for all } \omega \in \Omega\}$ which yields the induced background model $\mathcal{G} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0 \text{ for all } \omega \in \Omega\}$.

Deductive closure. The deductively closed assessment $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ is determined by

where $\mathcal{A} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0 \text{ for all } \omega \in \Omega\}$. The filter base generated by \mathcal{A}

No Confusion. is equivalent with the Dele construction property

No Limits. The reducing extension $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ is given by

The last two background model, Dele function, which means that the assessment/practical certainty is not in a state. Therefore, these extensions are not considered.

All practical certain events. Every the inference procedure described in the next section can be used for the set of all desirable gambles, or the larger set of desirable gambles \mathcal{A} . After any reasoning to infer a new inference procedure applied. For which events $\omega \in \Omega$ is $\omega \in \Omega$?

where $\mathcal{A} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0 \text{ for all } \omega \in \Omega\}$ is the filter generated by \mathcal{A} .

This tells us that we can specify interpretation of Ω , the logic of practical certainty that the active basic machinery on classical propositional logic.

Accept & reject statements

Accepting & rejecting. The subject gives his assessment \mathcal{A} by making accept and reject statements about gambles $g \geq 0$.

Accepting $g \geq 0$. implies a commitment for the subject to engage in the following transaction:
 (1) He will value $g(\omega)$ if ω is observed.
 (2) He will give the position negative payoff if ω is observed.
Rejecting $g \geq 0$ means that the subject indicates g from being accepted.

Assessment \mathcal{A} is a pair of accepted (\mathcal{A}) and rejected (\mathcal{R}) gambles: $\mathcal{A} = \{a, r\}$.

Second rationality requirement:

No Confusion. Given the integration attached to an accept and/or a reject statement, we have as a third rationality requirement:

There are four rationality requirements.

Background model. Before an assessment is given, some gambles can be guaranteed to be accepted and others to be rejected. Such a priori assumptions can be captured by posting a background model \mathcal{P} .

For rationality requirement:

No Limits. One can still accept or reject each (reduced) gamble $g \geq 0$ in $\mathcal{G} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0\}$. Gambles Lines $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ and $\mathcal{R} = \{r : \Omega \rightarrow \mathbb{R} \mid r(\omega) \geq 0\}$ in the positive scale that of \mathcal{G} . This observation is summarized at the reducing extension

Fourth rationality requirement:

Indifference. A gamble g is desirable if $g \geq 0$ and $g \geq 0$ is not in \mathcal{A} and its negation $\neg g$ is not accepted.

Assessment about one event ω . If a subject is practically certain that an event ω occurs, we will not take this to mean that he is not practically certain to win a sequentially, $\omega \in \Omega$ and $\omega \notin \Omega$.

Assessment about three events $\omega_1, \omega_2, \omega_3$. If he is practically certain that each event ω_i occurs, then his assessment is

No Confusion. it is only if $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$.

No Limits. Let $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ and $\mathcal{R} = \{r : \Omega \rightarrow \mathbb{R} \mid r(\omega) \geq 0\}$. The reducing extension $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ is given by

The last two background model, Dele function, which means that the assessment/practical certainty is not in a state. Therefore, these extensions are not considered.

All practical certain events. To find at each event, we use the set of $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ for which we have that $\omega \in \Omega$ if and only if $\omega \in \Omega$.

which leads to the same conclusions as for the first type of assessment.

Indifference assessments

Consider the collection of all assessments by \mathcal{A} . Consider the family of models for practical certainty following from the background model \mathcal{P} .

Then we have

meaning that $\mathcal{A} \subseteq \mathcal{A}$ is a complete belief when \mathcal{A} takes the role of \mathcal{A} in the background model \mathcal{P} .

For any $g \geq 0$ in \mathcal{G} , we have $g \geq 0$ and $g \geq 0$ is not in \mathcal{A} if and only if $g \geq 0$ is not in \mathcal{A} and $g \geq 0$ is not in \mathcal{A} . We have also

which means that \mathcal{A} and \mathcal{A} are essentially the same.

Consider the family of models for practical certainty following from background assessments.

$\mathcal{A} = \{g : \Omega \rightarrow \mathbb{R} \mid g(\omega) \geq 0\}$ and $\mathcal{R} = \{r : \Omega \rightarrow \mathbb{R} \mid r(\omega) \geq 0\}$.

Understandability $\mathcal{A} \subseteq \mathcal{A}$ is the only belief structure:

Finally, we can still take an understanding of Ω into account. Consider a reduced set of background gambles \mathcal{A} . We need first an assessment that is acceptable. If \mathcal{A} is not empty, $\mathcal{A} \subseteq \mathcal{A}$ must hold that $\mathcal{A} \subseteq \mathcal{A}$ and $\mathcal{A} \subseteq \mathcal{A}$. Let $\mathcal{A} = \{a : \Omega \rightarrow \mathbb{R} \mid a(\omega) \geq 0\}$ and $\mathcal{R} = \{r : \Omega \rightarrow \mathbb{R} \mid r(\omega) \geq 0\}$. Then

Accept & reject statements

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There are four rationality criteria.

- ▶ Indifference to status quo
- ▶ Deductive closure
- ▶ No Confusion
- ▶ No Limbo

Accept & reject statements

Accepting & rejecting. The subject gives an assessment of φ by making **accept** and **reject** statements about gambles (φ, β) .

Accepting (φ, β) . implies a commitment for the subject to engage in the following transaction:
Accepting (φ, β) . means that the subject includes β from being accepted.
Assessment α' is a pair of accepted (φ, β) and rejected (φ, γ) gambles: $\alpha' = (\varphi, \beta, \gamma)$.

Second rationality requirement:
 Given the integration attached to an assessment α' in a report statement, we have as a **third rationality requirement:**

No Confusion. Given the integration attached to an assessment α' in a report statement, we have as a **third rationality requirement:**

No Limbo. One can still accept or reject each **proposed** gamble $(\varphi, \beta) = (\varphi, \beta_1, \beta_2)$. Gambles (φ, β_1) and (φ, β_2) can **only** be rejected. If **No Confusion** is to be satisfied, then $\beta_1 = \beta_2 = \beta$. A gamble (φ, β) will be the positive scalar hull of β' . This observation is summarized in the following extension:

Fourth rationality requirement:

Deductive closure. If α' and α'' are assessments, then α' should be (φ, β) and α'' with $\beta = \beta_1 \cup \beta_2$. Thus, two observations are considered in the deductive extension.

Indifference. A gamble β is indifferent to β' if $\beta = \beta' \cup \beta''$ and β'' is rejected. If an subject is practically certain that an event α occurs, we will show how this is captured by a subject's responses to β and β' .

Assessment about state events. If β is an assessment about state events α , then β is an assessment about α if $\beta = \beta_1 \cup \beta_2$ and β_2 is rejected.

Background model. Because $\beta = \beta_1 \cup \beta_2$ is the result of this assessment, we can describe a slight extension of the background model β . This yields the **deductive extension** β' and β'' .

Deductive closure. The deductively closed assessment β' is $\beta' = \beta \cup \beta''$ and β'' is rejected.

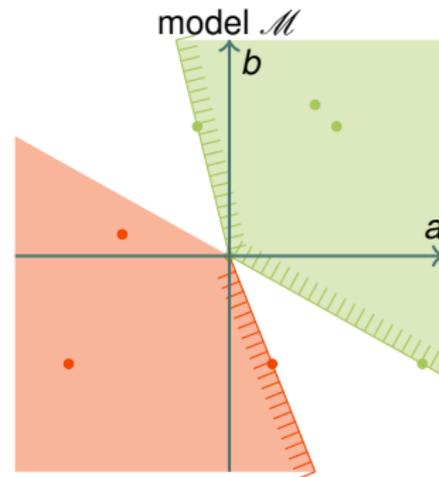
No Confusion. is equivalent with the **Stieglitz transitivity property**.

No Limbo. The following extension β' is given by:

Indifference assessments. Consider the extension β' of an assessment β . We will show how this is captured by an assessment β' following from the third rationality requirement.

Favourability assessments. Consider the family of reports for practical certainty below the first favourability assessment:

Practical certainty events. One can still accept or reject each **proposed** gamble $(\varphi, \beta) = (\varphi, \beta_1, \beta_2)$. Gambles (φ, β_1) and (φ, β_2) can **only** be rejected. If **No Confusion** is to be satisfied, then $\beta_1 = \beta_2 = \beta$. A gamble (φ, β) will be the positive scalar hull of β' . This observation is summarized in the following extension:



We can derive other sets of gambles.

Accept & reject statements

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Context. A subject who is practically certain about the occurrence of some event is a collection \mathcal{G} of gambles. We try to model this certainty in the form of an accept and reject statement based on expected utility. Motivation: This language is rich enough to encompass the abundant literature on rationality. In order to obtain more insight, three approaches are studied: different types of assessments, clarity of events, and the inclusion of a background model.

Assessment. This involves variables \mathcal{A} about which the subject has precise practical certainty. These events in \mathcal{G} are events in collection \mathcal{A} in the sense that $\forall A \in \mathcal{A}, \exists g \in \mathcal{G}, g(A) = 0$ and g is linear when restricted to the support of A . If $A \in \mathcal{A}$, then also $\exists g \in \mathcal{G}, g$ is linear on the support of A and $g(A) = 1$ and g is constant on the support of A . If $A \in \mathcal{A}$, then also $\exists g \in \mathcal{G}, g$ is linear on the support of A and $g(A) = 0$ and g is constant on the support of A . We denote the set of all gambles that are linear on all gambles in \mathcal{A} by $\mathcal{G}_{\mathcal{A}}$. We write $\mathcal{G} \subseteq \mathcal{G}_{\mathcal{A}}$ if $\forall g \in \mathcal{G}, g \in \mathcal{G}_{\mathcal{A}}$. The set of all such gambles is $\mathcal{G}_{\mathcal{A}}$. We write $\mathcal{G} \subseteq \mathcal{G}_{\mathcal{A}}$ if $\forall g \in \mathcal{G}, g \in \mathcal{G}_{\mathcal{A}}$.

Accepting & rejecting. The subject gives his assessment of \mathcal{A} by making accept and reject statements about gambles $(\mathcal{A}, \mathcal{P}, \mathcal{R})$.

Accepting \mathcal{A} . implies a commitment for the subject to sign in the following transaction:
 - Give actual value of \mathcal{P} if \mathcal{A} is determined.
 - If he gets the \mathcal{P} -portion (negative \mathcal{P} -part) if \mathcal{A} is determined.
Rejecting \mathcal{A} . implies that the subject is ready to sign being Accepted.
Assessment \mathcal{A} is a pair of accepted \mathcal{A} and rejected \mathcal{A} gambles $\mathcal{A} = (\mathcal{A}^+, \mathcal{A}^-)$.

Second rationality requirement:
 $\mathcal{G}_{\mathcal{A}} \cap \mathcal{G}_{\mathcal{A}^+} \cap \mathcal{G}_{\mathcal{A}^-} \neq \emptyset$

No Certainty. Given the information attached to an accept and a reject statement, we have as a **Third rationality requirement:**
 $\mathcal{G}_{\mathcal{A}^+} \cap \mathcal{G}_{\mathcal{A}^-} \neq \emptyset$

Then are four rationality requirements.

Background model. Before an assessment is given, some gambles can be perceived to be accepted and others to be rejected. Such a prior assumption can be captured by posting a background model \mathcal{M} .

For rationality requirement:
 $\mathcal{G}_{\mathcal{A}^+} \cap \mathcal{G}_{\mathcal{A}^-} \cap \mathcal{G}_{\mathcal{M}} \neq \emptyset$

No Limits. One can still accept or reject some constructed gambles $\mathcal{A} = (\mathcal{A}^+, \mathcal{A}^-)$. Gambles Linear $\mathcal{A} = (\mathcal{A}^+, \mathcal{A}^-)$, $\mathcal{A}^+ \subseteq \mathcal{A}^-$ can only be rejected. If No Certainty it is to be possible to accept \mathcal{A} . The observation is summarized in the following statement:
 $\mathcal{A} = \text{neg}(\mathcal{A}^+ \cap \mathcal{A}^-)$

Fourth rationality requirement:
 $\mathcal{G}_{\mathcal{A}^+} \cap \mathcal{G}_{\mathcal{A}^-} \cap \mathcal{G}_{\mathcal{M}} \neq \emptyset$

Deductive closure. If \mathcal{P} and \mathcal{R} are acceptable, then \mathcal{A} should be \mathcal{P} and \mathcal{R} with $\mathcal{A} \subseteq \mathcal{R}$. These two conditions are summarized in the deductive extension:
 $\mathcal{P} = \text{acc}(\mathcal{A} \cup \text{gen}(\mathcal{A}^+))$,
where $\text{gen}(\mathcal{A}^+) = \{ \mathcal{A} \cup \mathcal{A}^+ \mid \mathcal{A} \in \mathcal{R}, \mathcal{A} \subseteq \mathcal{A}^+ \}$, **the positive linear hull of \mathcal{A}^+ .**

Indifference. A gamble \mathcal{A} is indifferent if $\mathcal{A}^+ = \mathcal{A}^- = \mathcal{A}$ and $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$.
Assessment about one event. If a subject is practically certain that an event \mathcal{A} occurs, we will now try to model this fact by incorporating \mathcal{A} and \mathcal{A} in an assessment.

Assessment about one event \mathcal{A} . If the event \mathcal{A} occurs, then we will accept \mathcal{A} if $\mathcal{A} \subseteq \mathcal{P}$ and reject \mathcal{A} if $\mathcal{A} \subseteq \mathcal{R}$.

No Certainty. It is not only $\mathcal{A} \subseteq \mathcal{P}$.

No Limits. Let $\mathcal{A} = (\mathcal{A}^+, \mathcal{A}^-)$ with $\mathcal{A}^+ = \mathcal{A}^- = \mathcal{A}$ and $\mathcal{A} \subseteq \mathcal{R}$.

Background model. Because $\mathcal{A}^+ = \mathcal{A}^- = \mathcal{A}$, the notion of this assessment leads us to describe a slightly different background model $\mathcal{M} = (\mathcal{M}^+, \mathcal{M}^-)$. This yields the extended assessment $\mathcal{A} = (\mathcal{A}^+, \mathcal{A}^-)$ and $\mathcal{M} = (\mathcal{M}^+, \mathcal{M}^-)$.

Deductive closure. The deductively closed assessment $\mathcal{A} = (\mathcal{A}^+, \mathcal{A}^-)$ and $\mathcal{M} = (\mathcal{M}^+, \mathcal{M}^-)$ is determined by the induced gambles $\mathcal{G} = (\mathcal{G}^+, \mathcal{G}^-)$ with $\mathcal{G}^+ = \mathcal{A}^+ \cup \mathcal{M}^+$ and $\mathcal{G}^- = \mathcal{A}^- \cup \mathcal{M}^-$.

Indifference assessments. Describe the collection of all assessments \mathcal{A} . Consider the family of models for practical certainty following from indifference assessments:
 $\mathcal{M} = \{ (\mathcal{A}^+, \mathcal{A}^-) \mid \mathcal{A}^+ = \mathcal{A}^- = \mathcal{A} \}$.

Then we have:
meaning that $\mathcal{A} \subseteq \mathcal{R}$ is a complete lattice where \mathcal{A} plays the role of \mathcal{A}^+ and \mathcal{A}^- is an information state. But, for any $\mathcal{A} \in \mathcal{A}$, $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$ has the same power as \mathcal{A} is a gamble. $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$ are $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$ with $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$. We have $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$ which means that $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$ is exactly the same.

Favourability assessments. Describe the family of models for practical certainty following from favourability assessments:
 $\mathcal{M} = \{ (\mathcal{A}^+, \mathcal{A}^-) \mid \mathcal{A}^+ = \mathcal{A}^- = \mathcal{A} \}$.

Understandability $(\mathcal{A}, \mathcal{A}^-)$ is the following lattice structure:
 $\mathcal{A}^+ = \mathcal{A}^- = \mathcal{A}$

Finally, we can still find an interesting set of \mathcal{A} in \mathcal{A} . Consider a selected set of favourable gambles \mathcal{A} . We need first an assessment that includes \mathcal{A} and need $\mathcal{A} \subseteq \mathcal{P}$ with $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$. Let $\mathcal{A} = \mathcal{A}^+ = \mathcal{A}^- = \mathcal{A}$ with $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A} \subseteq \mathcal{P}$.

This tells us that an specific interpretation of \mathcal{A} , the logic of practical certainty from the above basic machinery on classical propositional logic.

We can derive other sets of gambles.

Accept & reject statements

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Context: A subject who is practically certain about the occurrence of some event is a collection \mathcal{G} of gambles.

Accept & reject statements: We try to model this certainty in the form of accept and reject statements based on subjective models.

Modeling: This language is rich enough to encompass the above mentioned concepts. In order to obtain more insight in these operations, we study different types of assessments.

Check all events: This involves checking if about which the subject is practically certain there must be \mathcal{F} . All events are covered in the sense that $\mathcal{G} = \{g_1, \dots, g_n\}$ and \mathcal{F} is the union of all events under these transactions (called *event composition*): $\mathcal{F} = \bigcup_{g \in \mathcal{G}} \text{supp}(g)$.

Check \mathcal{F} : \mathcal{F} is called *proper* if the addition $\mathcal{F} = \mathcal{F} \oplus \mathcal{F}$ is called *free* if $\mathcal{F} \oplus \mathcal{F}$ is closed under composition and \mathcal{F} is increasing under other (reducing) gambles $\mathcal{G}' = \mathcal{F} \oplus \mathcal{G}'$, i.e., $\mathcal{F} \oplus \mathcal{G}' \subseteq \mathcal{F}$ if $\mathcal{G}' \subseteq \mathcal{G}$.

Check all gambles: We describe the set of all gambles from \mathcal{F} (the set of gambles \mathcal{G}). A gamble g is described by the vector function $\mathcal{F} = \{g_1, \dots, g_n\}$. We describe the set of all gambles from \mathcal{F} (the set of gambles \mathcal{G}). We write $\mathcal{G} = \{g_1, \dots, g_n\}$. The set of all such gambles is $\mathcal{G} = \{g_1, \dots, g_n\}$. We write $\mathcal{G} = \{g_1, \dots, g_n\}$ if and only if the respective gambles $g_i = \mathcal{F}$.

Accepting & rejecting: The subject gives his assessment of by using accept and reject statements about gambles $f, g \in \mathcal{G}$.

Accepting f : implies a commitment for the subject to engage in the following transaction:
 - She will value u if \mathcal{F} is a statement
 - If the gain is $\text{—gain}(\text{negative—payoff})$ if \mathcal{F} occurs.
Rejecting f : means that the subject is not willing to be **Accepted**.

Assessment \mathcal{A} : is a pair of accepted (\mathcal{A}_+) and rejected (\mathcal{A}_-) gambles $\mathcal{A} = \{\mathcal{A}_+, \mathcal{A}_-\}$.

No Conclusion: Given the information attached to an asset and a reject statement, we have as a **Third rationality requirement:**

Then are four rationality requirements:

Background model: Before an assessment is given, some gambles can be purchased to be accepted and others to be rejected. Each a priori assumption can be captured by posting a background model \mathcal{M} .

For rationality requirement:

No Limits: One can still accept or reject each constructed gamble $g = \mathcal{M} \oplus \mathcal{F}$. Gambles from $\mathcal{M} \oplus \mathcal{F}$, $\mathcal{F} \subseteq \mathcal{G}$, can only be rejected, if \mathcal{F} is a statement. The observation is summarized as the following extension:
 $\mathcal{A} = \text{neg}(\mathcal{F}) = \{\mathcal{F}, \text{neg}(\mathcal{F}), \mathcal{F}, \mathcal{F}\}$.

Fourth rationality requirement:

Deductive closure: If \mathcal{A} and \mathcal{B} are acceptable, then $\mathcal{A} \cup \mathcal{B}$ should be \mathcal{A} and \mathcal{B} with $\mathcal{A} \cup \mathcal{B}$. These two operations are summarized in the deductive extension:
 $\mathcal{A} = \text{acc}(\mathcal{A} \cup \text{neg}(\mathcal{B})), \mathcal{B}$, where $\text{acc}(\mathcal{A}) = \{\mathcal{A}, \mathcal{A}\}$ and $\text{neg}(\mathcal{A}) = \{\mathcal{A}, \mathcal{A}\}$, the positive branch of \mathcal{A} .

Indifference: A gamble f is indifferent to g if $\mathcal{A} = \mathcal{A} \cup \text{neg}(\mathcal{F})$ and $\mathcal{B} = \mathcal{B} \cup \text{neg}(\mathcal{G})$ is an assertion.

Assessment about one event: If a subject is practically certain that an event \mathcal{A} occurs, we will have this to mean that he will not be interested in \mathcal{A} and \mathcal{A} is accordingly, $\mathcal{A} = \text{acc}(\mathcal{A})$.

Assessment about three events: If the probabilities within the set \mathcal{A} are equal, $\mathcal{A} = \text{acc}(\mathcal{A})$ if \mathcal{A} occurs, then $\mathcal{A} = \text{acc}(\mathcal{A})$.

No Conclusion: it will only if \mathcal{A} .

No Limits: Let $\mathcal{A} = \text{acc}(\mathcal{A}) \cup \text{neg}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B}) \cup \text{neg}(\mathcal{B})$ be the resulting extension of $\mathcal{A} = \text{acc}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B})$.

Background model: Because $\mathcal{A} = \text{acc}(\mathcal{A})$, the value of this assessment leads to a strictly defined background model $\mathcal{M} = \text{acc}(\mathcal{A})$. This yields the extended assessment $\mathcal{A} = \text{acc}(\mathcal{A}) \cup \text{neg}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B}) \cup \text{neg}(\mathcal{B})$.

Deductive closure: The deductively closed assessment $\mathcal{A} = \text{acc}(\mathcal{A}) \cup \text{neg}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B}) \cup \text{neg}(\mathcal{B})$ is determined by the induced gambles $\mathcal{A} = \text{acc}(\mathcal{A}) \cup \text{neg}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B}) \cup \text{neg}(\mathcal{B})$.

All practical certain events: Since the inference procedure described above, which relies only on the set of basic gambles, on the large set of favourable gambles \mathcal{A} , after any reasoning to inference it remains practically certain? For each event $\mathcal{A} \in \mathcal{A}$ it holds:

where $\mathcal{A} = \text{acc}(\mathcal{A}) \cup \text{neg}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B}) \cup \text{neg}(\mathcal{B})$ is the gamble generated by \mathcal{A} .

This rule is that an specific interpretation of \mathcal{A} , the logic of practical certainty from the above basic machinery to classical propositional logic.

Indifference assessments:

Describe the collection of all assessments \mathcal{A} . Consider the family of models for practical certainty following from indifference assessments:
 $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4, \mathcal{M}_5, \mathcal{M}_6, \mathcal{M}_7, \mathcal{M}_8, \mathcal{M}_9, \mathcal{M}_{10}\}$.

Then we have:

meaning that $\mathcal{A} \cup \mathcal{B}$ is a complete lattice where \mathcal{A} plays the role of $\mathcal{A} \cup \mathcal{B}$ as an intermediate element. For any $\mathcal{A} \in \mathcal{A}$ and $\mathcal{B} \in \mathcal{B}$, we have $\mathcal{A} \cup \mathcal{B} \in \mathcal{A}$ and $\mathcal{A} \cup \mathcal{B} \in \mathcal{B}$. We have $\mathcal{A} \cup \mathcal{B} \in \mathcal{A}$ and $\mathcal{A} \cup \mathcal{B} \in \mathcal{B}$.

which means that \mathcal{A} and \mathcal{B} are respectively the same.

Favourability assessments:

Describe the family of models for practical certainty following from favourability assessments:
 $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4, \mathcal{M}_5, \mathcal{M}_6, \mathcal{M}_7, \mathcal{M}_8, \mathcal{M}_9, \mathcal{M}_{10}\}$.

Understanding $\mathcal{A} \cup \mathcal{B}$ is the resulting lattice structure:

which we can see that that an interesting of \mathcal{A} and \mathcal{B} . Consider a selected set of favourable gambles \mathcal{A} , we need that an assessment from \mathcal{A} is not rejected. We can see that $\mathcal{A} \cup \mathcal{B}$ is not rejected. Let $\mathcal{A} = \text{acc}(\mathcal{A}) \cup \text{neg}(\mathcal{A})$ and $\mathcal{B} = \text{acc}(\mathcal{B}) \cup \text{neg}(\mathcal{B})$ be the gambles generated by \mathcal{A} and \mathcal{B} .

We can derive other sets of gambles.

- ▶ A gamble f is favourable if $f \in \mathcal{M}_\triangleright := \mathcal{M}_\geq \cap -\mathcal{M}_\prec$.

Accept & reject statements

Modelling practical certainty and its link with classical propositional logic
 Arthur Van Camp and Gert de Cooman
 SYSTEMS research group, Ghent University, Belgium



Context: A subject who is practically certain about the occurrence of some event in a collection Ω accepts a gamble f if and only if the net amount of money and resources based on this gamble is non-negative. Motivation: The language is rich enough to encompass the different types of gambles. The subject is not risk averse, more precisely, she does not have any preference for gambles.

Accepting & rejecting: The subject gives his assessment of f by saying accept or reject statements about gambles f, g, \dots . **Accepting f :** implies a commitment for the subject to engage in the following transaction: (1) He gives the positive (negative - payoff) f . **Rejecting f :** means that the subject includes f from being accepted. **Assessment of f :** is a pair of accepted (\succ) and rejected (\prec) gambles: $\mathcal{A} = \{a, r\}$.

Second rationality requirement: If the gamble f is accepted, then the gamble $f + g$ is also accepted. This is captured by the following constraint: $\mathcal{A} \cup \{g\} \subseteq \mathcal{A}$.

No Contradiction: Given the information attached to an asset and a reject statement, we have as a **Third rationality requirement:** $\mathcal{A} \cap \{g\} = \emptyset$.

There are four rationality requirements: Before an assessment is given, some gambles can be purchased to be accepted and others to be rejected. Each a priori assumption can be captured by posting a background model \mathcal{M} .

For rationality requirement: $\mathcal{M} \cap \mathcal{A} = \emptyset$.

No Limits: One can still accept or reject each unrestricted gamble $f \in \mathcal{M}$. Gambles $f \in \mathcal{M}$ can only be rejected, if No Contradiction is to be satisfied. This is captured by the following constraint: $\mathcal{A} \cap \mathcal{M} = \emptyset$.

Fourth rationality requirement: $\mathcal{M} \cap \mathcal{A} = \emptyset$.

Deductive closure: If f and g are accepted, then $f + g$ should be \succ and A, B with $A \cup B = \Omega$. These two constraints are summarized in the deductive extension $\mathcal{A}^d = \text{acc}(\mathcal{A}) \cup \text{rej}(\mathcal{A})$, where $\text{acc}(\mathcal{A}) = \{ \sum_{f \in \mathcal{A}} f, \sum_{f \in \mathcal{A}} f + g, \dots \}$, the positive linear hull of \mathcal{A} .

Indifference: A gamble f is indifferent if $f \in \mathcal{M} \cap \mathcal{A}^d$. Indifference about the event E is captured by practically certain that with a certain asset, we will lose this to not lose it in a non-trivial way, i.e., $\mathcal{A} \cap \mathcal{M} = \emptyset$.

Assessment about assets: If the asset f is not accepted, then f is not rejected. If f is not rejected, then f is not accepted. $\mathcal{A} \cap \mathcal{M} = \emptyset$.

No Contradiction: $\mathcal{A} \cap \mathcal{M} = \emptyset$.

No Limits: Let $\mathcal{M} = \{f, g, \dots\}$ be a gamble, then $f \in \mathcal{M}$ can only be rejected, if No Contradiction is to be satisfied. This is captured by the following constraint: $\mathcal{A} \cap \mathcal{M} = \emptyset$.

Deductive closure: The deductively closed assessment $\mathcal{A}^d = \text{acc}(\mathcal{A}) \cup \text{rej}(\mathcal{A})$ is determined by \mathcal{A} .

All practical certain events: Since the inference procedure described above, which relies only on the set of basic gambles, on the larger set of favourable gambles, after any reasoning to inference is closed, practical certainty for f is $f \in \mathcal{M} \cap \mathcal{A}^d$.

Indifference assessments: Describe the collection of all assessments \mathcal{A} . Consider the family of models for practical certainty following from indifference assessments: $\mathcal{M} = \{f, g, \dots\}$.

Favourability assessments: Describe the family of models for practical certainty following from favourability assessments: $\mathcal{M} = \{f, g, \dots\}$.

We can derive other sets of gambles.

- ▶ A gamble f is **favourable** if $f \in \mathcal{M}_{\triangleright} := \mathcal{M}_{\succeq} \cap -\mathcal{M}_{\prec}$.
- ▶ A gamble f is **indifferent** if $f \in \mathcal{M}_{\simeq} := \mathcal{M}_{\succeq} \cap -\mathcal{M}_{\succeq}$.

First type: favourability
assessment

Second type: indifference
assessment

Second type: indifference assessment

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 Arthur Van Camp and Gert de Cooman
 SYSTMS research group, Ghent University, Belgium



Context: A subject who is practically certain about the occurrence of some event ω is described by \mathcal{A} .
Accept & reject statements: We try to model this certainty in the language of accept and reject statements based on assessments models.
Modeling: The language is rich enough to accommodate the above mentioned model. In order to obtain more insight in these operations, we study different types of assessments.
Practical certainty: This model involves a total order on the power set of all events $\mathcal{A} = \{A \mid A \subseteq \Omega\}$ and a filter \mathcal{F} is induced when these assessments (called *practical certainty*) $A \in \mathcal{A}$ are $\omega \in A$, then also $\omega \in B$ if ω is called *practical* for the addition $A \cup B$ of A and B .
Practical certainty: \mathcal{A} is called *practical* and \mathcal{F} is accordingly called *practical* (inductively) if $\mathcal{A} \cup \mathcal{F}$ and $\mathcal{A} \cap \mathcal{F}$ are called *practical* if \mathcal{F} is defined by the set of all ranges from \mathcal{A} to \mathcal{A} of all generators $A \in \mathcal{A}$. A generator is described by the relative function $\omega \mapsto \omega \cap A$ of all generators $A \in \mathcal{A}$. We write $\omega \in A$ if $\omega \in A$ and $\omega \in B$ if $\omega \in B$ for all $A, B \in \mathcal{A}$. We write $\omega \in A \cup B$ if $\omega \in A$ or $\omega \in B$ and $\omega \in A \cap B$ if $\omega \in A$ and $\omega \in B$. The set of all such generators is \mathcal{A}^* . $\mathcal{A}^* \cup \mathcal{F}^* = \mathcal{A} \cup \mathcal{F}$ and $\mathcal{A}^* \cap \mathcal{F}^* = \mathcal{A} \cap \mathcal{F}$.

Assessments: A generator A is *desirable* if $\omega \in A$ or $\omega \in A^*$ if ω is desirable and $\omega \in A^*$.
Assessments about an event: If a subject is practically certain that an event ω occurs, we will not care for the fact that he holds any given assessment $A \in \mathcal{A}$ if $\omega \in A$. We will not care for the fact that he holds any assessment $A \in \mathcal{A}$ if $\omega \in A^*$. We will not care for the fact that he holds any assessment $A \in \mathcal{A}$ if $\omega \in A^*$. We will not care for the fact that he holds any assessment $A \in \mathcal{A}$ if $\omega \in A^*$.

Background model: We assume the background model $\mathcal{A}^* = \{A \cup A^* \mid A \in \mathcal{A}\}$ which yields the smallest assessment \mathcal{A}^* that is suitable for \mathcal{A} .

Deductive closure: The deductively closed assessment $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$ is inductively closed.
 when $\omega \in A \cup B$ and $\omega \in A \cap B$, the filter base generated by \mathcal{A} .

No Contradiction: is equivalent with the filter intersection property.
No Limits: The inductively closed $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$ is given by $\mathcal{A}^* = \{A \cup A^* \mid A \in \mathcal{A}\}$.
 The list has examples from Contradictions. Therefore, these examples are not continued.

All practical certain events: Shows the inference procedure described above, which yields a set \mathcal{A}^* from the set of basic generators. In the larger set of desirable generators \mathcal{A}^* , later any remaining to inference is considered *practical* (inductively) for \mathcal{A} and \mathcal{A}^* .
 when $\mathcal{A}^* = \{A \cup A^* \mid A \in \mathcal{A}\}$ is the filter generated by \mathcal{A} .
 This sets us that we can specify interpretation of \mathcal{A} , the logic of practical certainty from the above basic machinery on classical propositional logic.

Accepting & rejecting: The subject gives his assessment ω by making accept and reject statements about generators $A \in \mathcal{A}$.
Accepting ω : implies a contribution for the subject to an event ω . The actual value of ω is determined by the plus (or minus) region (accept) $\omega \in A$.
Rejecting ω : implies that the subject indicates ω is not being accepted.
Assessment ω : is a pair of accepted $\omega \in A$ and rejected $\omega \in B$ generators $A \in \mathcal{A}$ and $B \in \mathcal{A}$.
Second rationality requirement:
No Contradiction: Given the interpretation described in an assessment model, we have to have a rationality requirement.
Third and fourth rationality requirements:
Background model: Before an assessment ω is given, basic generators can be generated to be accepted and others to be rejected. As a pair assessment can be captured by pointing a background model.
First rationality requirement:
No Limits: One can still assign or reject some assessment generators $A \in \mathcal{A}$ and $B \in \mathcal{A}$.
Fourth rationality requirement:
Deductive closure: If ω and ω are acceptable, then ω should be $\omega \in A$ and $\omega \in B$. These two conditions are not mentioned in the deductive extension $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$ where $\omega \in A \cup B$ and $\omega \in A \cap B$, the problem comes that \mathcal{A}^* .

Second type: indifference assessment

Indifference: A generator (or evaluation) $A \in \mathcal{A}$ is acceptable, $\omega \in A$ and $\omega \in A^*$ if $\omega \in A$ or $\omega \in A^*$ if ω is acceptable.

Assessment about an event: If a subject is practically certain that an event ω occurs, we will not take this to mean that he is indifferent between A and A^* or equivalently between A and B .

Assessment about these events ω : If he is practically certain that each event $\omega \in A$ occurs, then he is indifferent between A and A^* .

Background model: Because $A \in \mathcal{A}$ and $A^* \in \mathcal{A}^*$, the value of the assessment bases is to include a slightly different background model $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$. This yields the smallest assessment $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$ that includes \mathcal{A} and \mathcal{A}^* .

Deductive closure: The deductively closed assessment $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$ is inductively closed.
 when $\omega \in A \cup B$ and $\omega \in A \cap B$, the filter base generated by \mathcal{A} .

No Contradiction: is equivalent with the filter intersection property.
No Limits: The inductively closed $\mathcal{A}^* = \mathcal{A} \cup \mathcal{A}^*$ is given by $\mathcal{A}^* = \{A \cup A^* \mid A \in \mathcal{A}\}$.
 The list has examples from Contradictions. Therefore, these examples are not continued.

All practical certain events: To find all such events, we look at the events $A \in \mathcal{A}$ for which we have that $\omega \in A$ or $\omega \in A^*$.
 which leads to the same conclusions as for the first type of assessments.

Indifference assessments:
 Describe the construction of all assessments \mathcal{A} . Consider the family of models for practical certainty following from indifference assessments.
 when $\mathcal{A}^* = \{A \cup A^* \mid A \in \mathcal{A}\}$ is a filter base.
 meaning that $\mathcal{A} \cup \mathcal{A}^*$ is a complete lattice where \mathcal{A} plays the role of \mathcal{A} and \mathcal{A}^* is an assessment structure. For any $A \in \mathcal{A}$ and $A^* \in \mathcal{A}^*$, we have $\omega \in A$ and $\omega \in A^*$ if $\omega \in A$ or $\omega \in A^*$. We have also $\omega \in A \cup A^*$ if $\omega \in A$ or $\omega \in A^*$.
 which means that $\mathcal{A} \cup \mathcal{A}^*$ is inductively the same.

Verifiability assessments:
 Consider the family of models for practical certainty following from verifiability assessments.
 when $\mathcal{A}^* = \{A \cup A^* \mid A \in \mathcal{A}\}$ is a filter base.
 verifiability $\mathcal{A} \cup \mathcal{A}^*$ is a filtering lattice structure.
 finally, we can still find an interesting set of \mathcal{A} and \mathcal{A}^* . Consider a set of all desirable generators \mathcal{A} . We need that an assessment from \mathcal{A} is called *practical* if $\omega \in A$ and $\omega \in A^*$ if $\omega \in A$ or $\omega \in A^*$. We have also $\omega \in A \cup A^*$ if $\omega \in A$ or $\omega \in A^*$.
 which means that $\mathcal{A} \cup \mathcal{A}^*$ is inductively the same.

Second type: indifference assessment

Context: A subject who is practically certain about the occurrence of some event A is described by \mathcal{A} .
Assessing & reporting: We try to model the certainty in the background model \mathcal{B} by means of a set of assessment functions. This language is rich enough to accommodate the abundant information that the subject provides to allow more insight in his preferences, we study different types of assessments.
Inductive closure: The inductive closure \mathcal{A}^+ of \mathcal{A} is the set of all assessment functions that extend \mathcal{A} . It is the smallest set of assessment functions that contains \mathcal{A} and is closed under the operations of pointwise addition and multiplication. The inductive closure \mathcal{A}^+ is the set of all assessment functions \mathcal{A}^+ that can be obtained from \mathcal{A} by a finite number of applications of the operations of pointwise addition and multiplication. The set of all such functions is \mathcal{A}^+ .

Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .



Inductive closure: The inductively closed assessment \mathcal{A}^+ of \mathcal{A} is the set of all assessment functions that extend \mathcal{A} .
Assessment about three events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .



Assessing & reporting: The subject gives his assessment about A and B by means of a set of assessment functions \mathcal{A} and \mathcal{B} .
Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .

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Inductive closure: The inductively closed assessment \mathcal{A}^+ of \mathcal{A} is the set of all assessment functions that extend \mathcal{A} .
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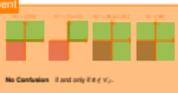
Assessing & reporting: The subject gives his assessment about A and B by means of a set of assessment functions \mathcal{A} and \mathcal{B} .
Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .

Indifference assessments: Consider the family of models for practical certainty following from indifference assessments \mathcal{A} .
Assessing & reporting: The subject gives his assessment about A and B by means of a set of assessment functions \mathcal{A} and \mathcal{B} .
Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .

Second inductively requirement: The subject gives his assessment about A and B by means of a set of assessment functions \mathcal{A} and \mathcal{B} .
Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .



Inductive closure: The inductively closed assessment \mathcal{A}^+ of \mathcal{A} is the set of all assessment functions that extend \mathcal{A} .
Assessment about three events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .



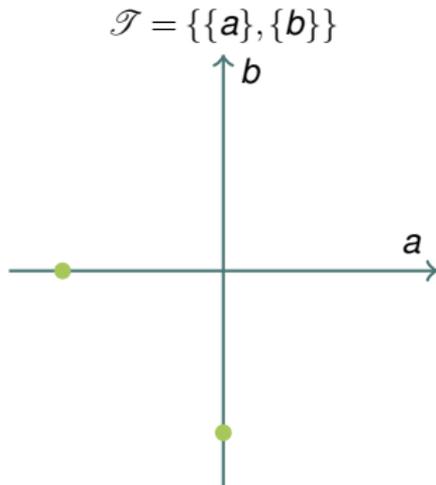
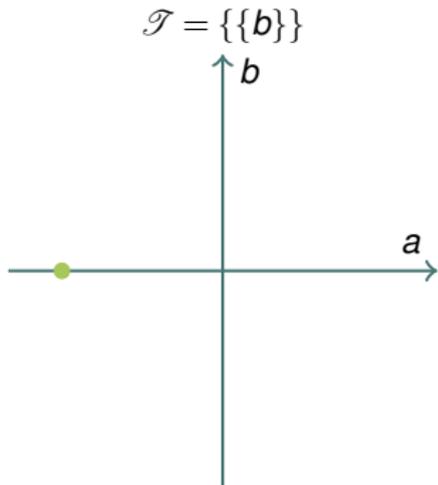
Assessing & reporting: The subject gives his assessment about A and B by means of a set of assessment functions \mathcal{A} and \mathcal{B} .
Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .

Indifference assessments: Consider the family of models for practical certainty following from indifference assessments \mathcal{A} .
Assessing & reporting: The subject gives his assessment about A and B by means of a set of assessment functions \mathcal{A} and \mathcal{B} .
Assessing: A gamble \mathcal{G} is assessable if $\mathcal{G} \in \mathcal{A}^+$.
Assessment about one event: If a subject is practically certain that an event A will occur, we will not try to model this by any other gamble than \mathcal{A} .
Assessment about two events: If a subject is practically certain that both events A and B will occur, then the assessment \mathcal{A} is not sufficient to model this. We need to add to \mathcal{A} an assessment \mathcal{B} that is not in \mathcal{A}^+ .

If a subject is practically certain that an event A will occur, we will now take this to mean that he is **indifferent between \mathbb{I}_A and 1**, or equivalently, between \mathbb{I}_{A^c} and 0.
 $\Rightarrow \mathcal{A} \approx^A := \{\mathbb{I}_{A^c}\}$ is indifferent.
 $\Rightarrow \mathcal{A} \approx^A = \{\pm \mathbb{I}_{A^c}\}$ is acceptable.

Second type: indifference assessment

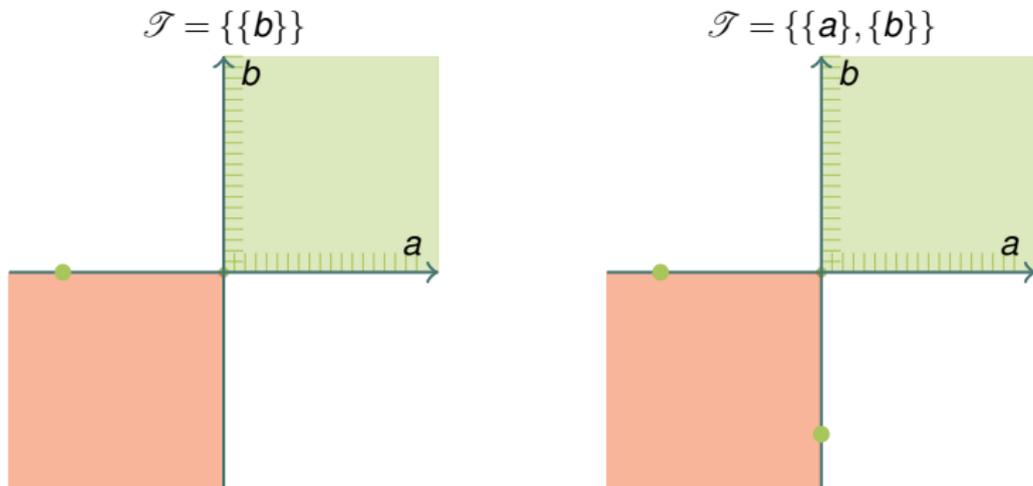
Assessment \mathcal{A}'



Second type: indifference assessment

Smallest assessment that includes the background model

$$\mathcal{B}' = \mathcal{A}' \cup \langle \mathcal{L}_{\geq 0}; \mathcal{L}_{< 0} \rangle$$

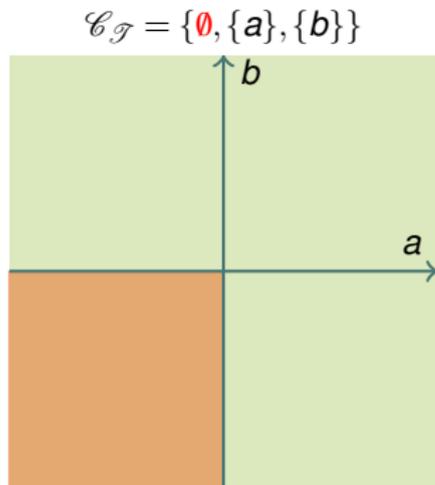
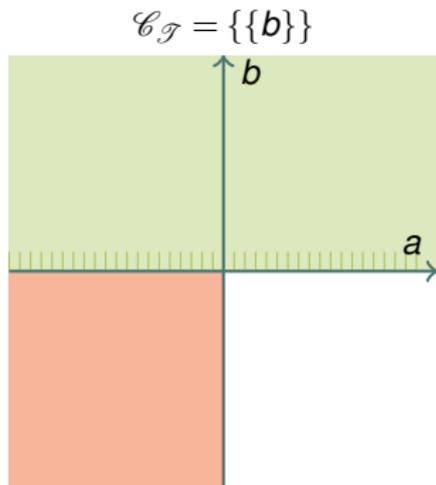


First rationality requirement Indifference to status quo: $0 \in \mathcal{L}_{\geq 0}$.

Second type: indifference assessment

Deductive closure

$$\mathcal{D}' = \langle \text{posi } \mathcal{B}'_{\succeq}; \mathcal{B}_{\prec} \rangle \text{ with } \text{posi } \mathcal{B}'_{\succeq} = \{f \in \mathcal{L} : (\exists B \in \mathcal{C}_{\mathcal{G}}) \mathbb{I}_B f \geq 0\}$$



Second rationality requirement: \mathcal{D}' should be Deductive Closed.

Second type: indifference assessment

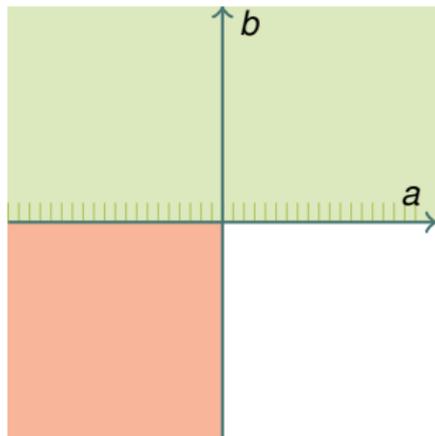
Deductive closure

$$\mathcal{D}' = \langle \text{posi } \mathcal{B}'_{\succeq}; \mathcal{B}_{\prec} \rangle \text{ with } \text{posi } \mathcal{B}'_{\succeq} = \{f \in \mathcal{L} : (\exists B \in \mathcal{C}_{\mathcal{G}}) \mathbb{I}_B f \geq 0\}$$

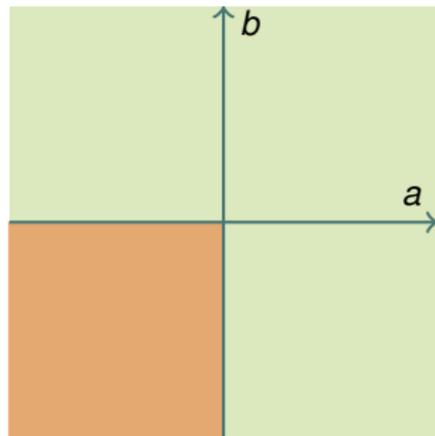
$$\text{Here, } \mathcal{C}_{\mathcal{G}} := \left\{ \bigcap_{k=1}^n \bar{A}_k : n \in \mathbb{N}, A_k \in \mathcal{T} \right\}$$

is the filter base generated by \mathcal{T} .

$$\mathcal{C}_{\mathcal{G}} = \{\{b\}\}$$



$$\mathcal{C}_{\mathcal{G}} = \{\emptyset, \{a\}, \{b\}\}$$



Second rationality requirement: \mathcal{D}' should be Deductive Closed.

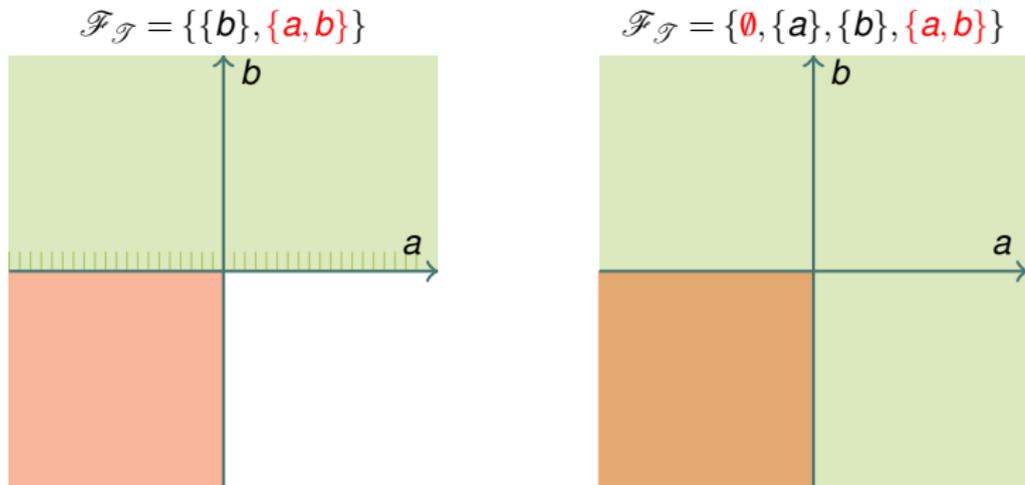
Second type: indifference assessment

Deductive closure

$$\mathcal{D}' = \langle \text{posi } \mathcal{B}'_{\succeq}; \mathcal{B}_{\prec} \rangle \text{ with } \text{posi } \mathcal{B}'_{\succeq} = \{f \in \mathcal{L} : (\exists B \in \mathcal{C}_{\mathcal{G}}) \mathbb{I}_B f \geq 0\}$$

Here, $\mathcal{F}_{\mathcal{G}} := \{B \in \mathcal{P} : (\exists C \in \mathcal{C}_{\mathcal{G}}) C \subseteq B\}$

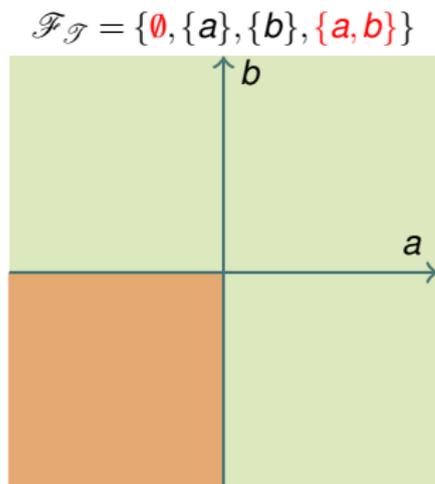
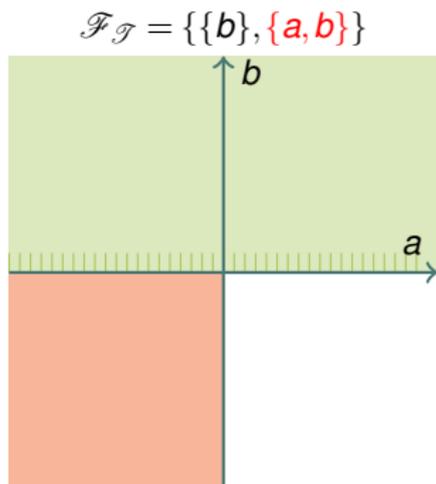
is the **filter** generated by \mathcal{T} .



Second rationality requirement: \mathcal{D}' should be Deductive Closed.

Second type: indifference assessment

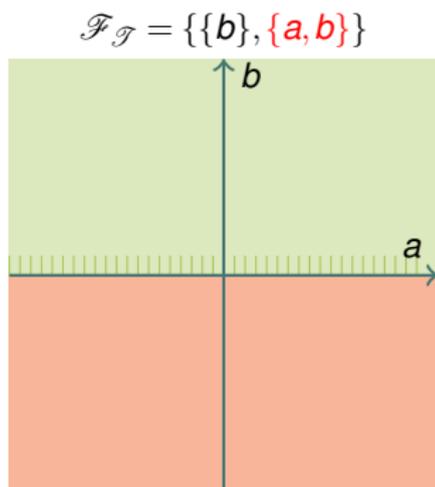
No Confusion: $\mathcal{D}'_{\Sigma} \cap \mathcal{D}'_{\Sigma} = \emptyset$



Third rationality requirement: **No Confusion** $\Leftrightarrow \emptyset \notin \mathcal{C}_{\mathcal{G}}$.

Second type: indifference assessment

No Limbo: $(\overline{\mathcal{D}}_{\prec} - \mathcal{D}_{\prec}) \setminus \mathcal{D}_{\prec}$ should be rejected



Fourth rationality requirement: **No Limbo.**

Embedding classical propositional logic

Embedding classical propositional logic

Modelling practical certainty and its link with classical propositional logic

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<p>Context \mathcal{F} subject who is practically certain about the occurrence of every element in a collection \mathcal{X}.</p> <p>Assess & report statements We try to model this certainty in the form of an <i>assessment</i> and <i>report statements</i> based on elementary events.</p> <p>Assessment The language is rich enough to encompass the different ways in which \mathcal{F} assesses \mathcal{X}. In order to obtain more insight in these operations, we study different types of assessments.</p> <p>Practical certainty There are two ways in which \mathcal{F} assesses \mathcal{X}. There are all events, the practical certainty states about \mathcal{X}. \mathcal{F} assesses the collection in the power set $\mathcal{P}(\mathcal{X}) = \{A \subseteq \mathcal{X} \mid A \text{ is a filter}\}$. \mathcal{F} also states what filter measurements (called <i>filters</i>) are possible. \mathcal{F} is <i>practically certain</i> if \mathcal{F} is closed under negation and \mathcal{F} is an <i>assessment</i> (called <i>total</i>) if \mathcal{F} is closed under conjunction. And \mathcal{F} is an <i>assessment</i> (called <i>total</i>) if \mathcal{F} is closed under conjunction and \mathcal{F} is a <i>filter</i> (called <i>total</i>) if \mathcal{F} is closed under negation. And \mathcal{F} is called <i>total</i> if \mathcal{F} is a subset of \mathcal{F}. We describe the set of all filters that \mathcal{F} is <i>practically certain</i> in \mathcal{F}.</p> <p>The set of all filters in \mathcal{F} is $\mathcal{F} = \{A \subseteq \mathcal{X} \mid A \text{ is a filter}\}$. We write \mathcal{F} and the set of all such filters in \mathcal{F}. We write $\mathcal{F} = \{A \subseteq \mathcal{X} \mid A \text{ is a filter}\}$. The set of all such filters in \mathcal{F} is $\mathcal{F} = \{A \subseteq \mathcal{X} \mid A \text{ is a filter}\}$. We write \mathcal{F} and the set of all such filters in \mathcal{F}.</p>	<p>Assessing & reporting The subject gives his assessment report \mathcal{F} by making reports and report statements about \mathcal{X}.</p> <p>Assessing \mathcal{F} makes a statement for the subject to be given in the following instruction.</p> <p>Reporting \mathcal{F} makes the subject's decision, by making a statement for the subject to be given in the following instruction.</p> <p>Assessment \mathcal{F} is a pair of accepted \mathcal{F} and rejected \mathcal{F} in \mathcal{F}.</p>	<p>Second reliability requirement</p> <p>No Conclusion Given the information obtained in an assessment report, we have a \mathcal{F} reliability requirement.</p>
<p>Assessing A filter \mathcal{F} is desirable if $\mathcal{F} = \{A \subseteq \mathcal{X} \mid A \text{ is a filter}\}$.</p> <p>Assessment about those events \mathcal{F} is practically certain that \mathcal{F} is a filter, and we can use this to show that the filter \mathcal{F} is a filter.</p> <p>Assessment about those events \mathcal{F} is practically certain that \mathcal{F} is a filter, and we can use this to show that the filter \mathcal{F} is a filter.</p>	<p>Background model Before an assessment is given, some filters can be preferred to be accepted and others to be rejected. Such a prior assessment can be captured by posing a background model.</p> <p>First reliability requirement</p> <p>No Limit One can still accept or reject some assessment reports in \mathcal{F}.</p> <p>Fourth reliability requirement</p>	<p>Background model Before an assessment is given, some filters can be preferred to be accepted and others to be rejected. Such a prior assessment can be captured by posing a background model.</p> <p>First reliability requirement</p> <p>No Conclusion Given the information obtained in an assessment report, we have a \mathcal{F} reliability requirement.</p>
<p>Background model We assume the background model $\mathcal{F} = \{A \subseteq \mathcal{X} \mid A \text{ is a filter}\}$. This is the model we use to show that the filter \mathcal{F} is a filter.</p>	<p>Definitive closure If \mathcal{F} and \mathcal{F} are assessments, then we should let $\mathcal{F} = \mathcal{F} \cup \mathcal{F}$, with $\mathcal{F} = \mathcal{F}$. These operations are summarized in the definitive closure.</p> <p>Assessment about those events \mathcal{F} is practically certain that \mathcal{F} is a filter, and we can use this to show that the filter \mathcal{F} is a filter.</p>	<p>Definitive closure If \mathcal{F} and \mathcal{F} are assessments, then we should let $\mathcal{F} = \mathcal{F} \cup \mathcal{F}$, with $\mathcal{F} = \mathcal{F}$. These operations are summarized in the definitive closure.</p> <p>Assessment about those events \mathcal{F} is practically certain that \mathcal{F} is a filter, and we can use this to show that the filter \mathcal{F} is a filter.</p>
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We show that the language of our models for practical certainty essentially equals the **language of filters**.

Embedding classical propositional logic

Indifference assessments Consider the collection of all assessments \mathcal{F} . A **filter** is a family of models for practical certainty following indifference assessments.

They are true \mathcal{F} is a strong belief structure, meaning that \mathcal{F} is a complete lattice where \mathcal{F} plays the role of atoms. \mathcal{F} is an implicative structure. For any $\mathcal{F}, \mathcal{F}' \in \mathcal{F}$, we have $\mathcal{F} \cup \mathcal{F}' \in \mathcal{F}$. We have also $\mathcal{F} \cap \mathcal{F}' \in \mathcal{F}$. We have also $\mathcal{F} \cup \mathcal{F}' \in \mathcal{F}$ and $\mathcal{F} \cap \mathcal{F}' \in \mathcal{F}$.

They are false \mathcal{F} is a strong belief structure, meaning that \mathcal{F} is a complete lattice where \mathcal{F} plays the role of atoms. \mathcal{F} is an implicative structure. For any $\mathcal{F}, \mathcal{F}' \in \mathcal{F}$, we have $\mathcal{F} \cup \mathcal{F}' \in \mathcal{F}$. We have also $\mathcal{F} \cap \mathcal{F}' \in \mathcal{F}$. We have also $\mathcal{F} \cup \mathcal{F}' \in \mathcal{F}$ and $\mathcal{F} \cap \mathcal{F}' \in \mathcal{F}$.

Favourability assessments Consider the family of models for practical certainty following favourability assessments.

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